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I. Vas
Associate Editor

Skin-Friction Prediction for High-Speed Turbulent Boundary Layers with Ablation

Yichuan Fang* and William W. Liou†
Western Michigan University, Kalamazoo, Michigan 49008
and
Shuxuan Xu‡
University of Science and Technology of China, 230026 Hefei, Anhui, People's Republic of China

Introduction

DEVELOPMENT of advanced high-speed missiles and reentry aerospace vehicles has prompted continued research in aerothermodynamics with increased emphasis on accurate predictions of their aerodynamic performance. In such applications, the effects of surface ablation have received considerable attention. Inherent in the ablation process is surface erosion and accompanying development of surface roughness and mass blowing. Surface roughness generally causes augmentation of the skin friction over aerodynamic surfaces, whereas blowing causes a reduction. In an ablation process, these two phenomena interact and their combined effects on skin friction are not well understood.

The effects of surface roughness on skin friction have been examined in many experiments and numerical calculations. Since Nikuradse's[1] work on sand-grain pipe flows, experiments on roughness effects have been reported.[1–4] The effects of roughness have been considered based on the standard of sand-grain roughness initially proposed by Schlichting.[2] Roughness elements were analyzed using equivalent sand-grain roughness. Many studies on the effects of mass blowing on skin friction have been reported for a wide range of flow speeds. The effects of molecular weight of the blowing gas, wall temperature, flow Mach number, angle of attack, and boundary-layer transition were addressed in various investigations.[5–8] The experimental results do not always match and, in some cases, the disagreement is quite significant. Lin et al.[9] contributed a large scattering of experimental data on effects of transverse curvature, pressure gradient, flow being not fully turbulent, different wall temperature ratios, Mach numbers, and how gases were injected from the surface. Jeromin[10] and Laganelli et al.[7] also pointed out that the effects of blowing on skin friction become more significant as the Mach number increases.

Many numerical simulations of ablation[11–14] have been reported recently. Issues related to the design of thermal protection systems (TPSs) for spacecraft and hypersonic reentry vehicles were examined by considering the thermal response of the material and mass injection. The surface roughness effects of ablation, especially on the skin friction and other aerodynamic variables, are not reported.

There are currently no theoretical analyses that can be used to evaluate skin friction under the combined influence of roughness and blowing. Voisinet[15] and Holden[16] reported experimental studies on the combined effect. Based on experimental observations, Voisinet[15] suggested a functional relationship to couple roughness effects to those of mass blowing. Laganelli and Sontowski[17] proposed a method that includes the effect of pressure gradient to predict skin friction with coupled roughness and blowing.

Inspired by the work of Laganelli and Sontowski[17] and the experimental findings of Voisinet,[15] we have developed a new computational methodology to calculate the combined effects of roughness and blowing on skin friction. The two-step approach involves iterative calculation of skin friction with roughness and a second explicit calculation using the information obtained in the previous step. The results of skin-friction calculations for a supersonic turbulent boundary layer over a flat plate will be presented and the results will be compared with available experimental data.

Technical Approach

Effect of Roughness on Skin Friction

Goddard[18] proposed a correlation of the form

$$c_f^{o/r} \big/ c_f^{o/o} = f(\lg Re_k) \qquad (1)$$

to account for roughness effects on skin friction for adiabatic no-blowing walls. The superscripts o/r and o/o represent without-blowing/with-roughness and without-blowing/without-roughness, respectively. Also,

$$Re_k = u_\tau k_s / \upsilon_w, \qquad u_\tau = u_e \sqrt{\left(c_f^{o/r}\big/2\right)(\rho_e/\rho_w)} \qquad (2)$$

k_s denotes the equivalent sand-grain roughness height. The functional form in Eq. (1) was not explicitly given in Goddard. A comparison with data (Fig. 27 in Goddard[18]), however, showed satisfactory agreement. Nestler[19] replotted Goddard's results using the logarithm to the base 10 of a roughness Reynolds number and included the effects of wall cooling.

In this study, a correlation has been developed to fit the data in Fig. 27 of Goddard.[18] The new correlation can be written as

$$c_f^{o/r} \big/ c_f^{o/o} = 1 + 0.889[0.365(T_w/T_r) + 0.635] \times \left(\lg Re_k - \lg Re_k^c\right)\left(Re_k > Re_k^c\right) \qquad (3)$$

Re_k^c represents a critical roughness Reynolds number, below which the skin friction is assumed not to be affected by the surface

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*Research Associate, Department of Mechanical and Aeronautical Engineering.

†Professor, Department of Mechanics and Mechanical Engineering.

‡Professor, Department of Mechanics and Mechanical Engineering; currently retired.

roughness. The value of Re_k^c used by Goddard[18] was about 10. The roughness Reynolds number is defined using the skin friction coefficient with roughness $c_f^{o/r}$. As a result, Eq. (3) maintains the implicit nature of Goddard's[18] formulation. Note that Nestler's[19] correction for nonadiabatic conditions has also been included.

It can be argued that the value of the critical roughness Reynolds number Re_k^c in Eq. (3) represents a measure for the minimum equivalent height of the roughness that has an effect on the skin friction. For example, the value of Re_k^c was set at about 10 in Goddard[18] (Mach number = 0.7–5) and 30 in Holden[16] (Mach number = 6–13). The difference in the values of Re_k^c used in the different studies can be attributed to the increased thickness of the viscous sublayer as the flow Mach number increases. To account for the apparent effect of flow speed on Re_k^c, we consider

$$Re_k^c = \begin{cases} 9 & (M \le 0.7) \\ 10 & (0.7 < M \le 3) \\ M+7 & (3 < M \le 12) \end{cases} \tag{4}$$

Equation (4) slightly modifies the single value for Re_k^c used in Goddard[18] (=10).

Effect of Blowing on Skin Friction

Mass blowing on the surface tends to reduce the momentum of gases, increase the boundary layer thickness, and reduce the wall skin friction. Empirical correlations have been proposed.[20–22] Laganelli et al.[7] reported a correlation that includes the effects of Mach number, wall temperature, and injection gas characteristics. These correlations differ significantly because they have been developed through considering the different factors. In the present modeling, the correlations of Timmer et al.[6] and Lees and Chapkis[8] were used.

Combined Effects of Roughness and Blowing on Skin Friction

Laganelli and Sontowski[17] proposed a coupling method based on transformation functions to calculate the skin friction. Voisinet[15] studied the combined effects of roughness and blowing on skin friction experimentally. Various parametric arrangements were used to correlate the experimental data. A better fit for the skin-friction data for a rough surface with blowing was observed when the skin friction with only the roughness present was used in a subsequent correction for the blowing effects. As a result, Voisinet[15] suggested an equation of the form

$$(c_f/c_{fo})|_{R+M} = (c_f/c_{fo})|_R (c_f/c_{fo})|_M \tag{5}$$

Voisinet's[15] experimental findings indicate that the skin friction for rough surfaces with blowing can be evaluated as the product of two terms, one representing that with roughness and the other with blowing, with proper coupling in the calculations of the two values.

In this study, a computational methodology has been devised for prediction of skin friction on rough surfaces with blowing. An equation similar to that suggested by Voisinet's[15] experiment is used:

$$c_f^{m/r}/c_f^{o/o} = \left(c_f^{m/r}/c_f^{o/r}\right)\left(c_f^{o/r}/c_f^{o/o}\right) \tag{6}$$

The computations involve the calculations of the two ratios of skin friction coefficients in Eq. (6). The second term on the right-hand-side, $c_f^{o/r}/c_f^{o/o}$, is first calculated. In this Note, Eqs. (3) and (4) are used for $c_f^{o/r}/c_f^{o/o}$. Note that Eq. (3) is implicit and the solution requires iterations based on given skin friction for a smooth wall. The next step is to obtain the first right-hand-side term of Eq. (6), $c_f^{m/r}/c_f^{o/r}$, for the mass-blowing effects. The skin-friction coefficient on the rough surface with blowing can then be obtained from Eq. (6).

Results

A new correlation for the skin friction for the boundary layer over rough surfaces has been proposed and described in Eqs. (3) and (4). Figure 1 shows a comparison of the proposed correlation with the adiabatic-wall data of Goddard.[18] The symbols represent the experimental measurement. The lines denote the proposed correlation

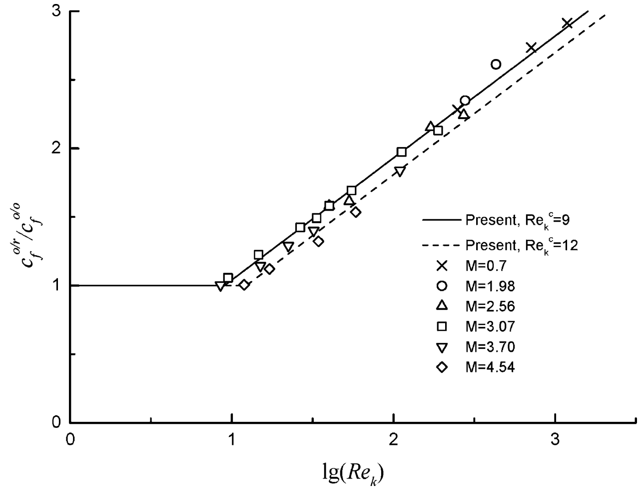

Fig. 1 Comparison of the proposed correlation with findings of Goddard[18] for a flat plate.

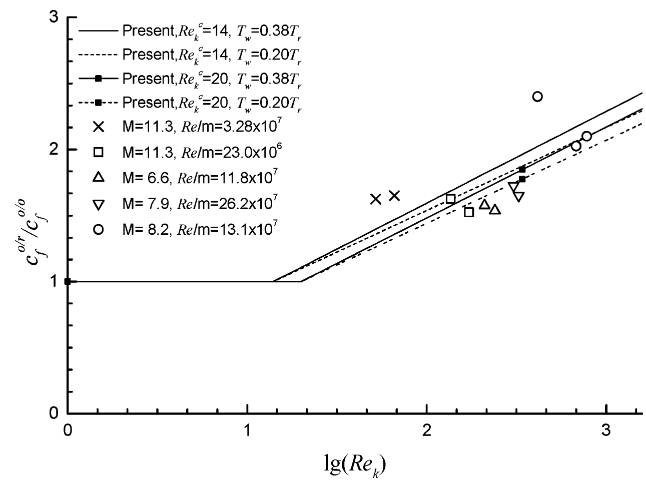

Fig. 2 Comparison of the roughness effect correlation with findings of Holden[16] for a sharp cone with a cold wall.

with $Re_k^c = 9$ for the solid line and $Re_k^c = 12$ for the dashed line, respectively. According to Eq. (4), these two values for Re_k^c correspond to the lowest (0.7) and the highest (4.54) Mach numbers in the experiment. The data points appear in the band region between the two lines with the low-Mach-number points nearing the upper solid line and the high-Mach-number data nearing the lower dashed line. The results suggest that Eqs. (3) and (4) correlate well with the variation of skin friction with wall roughness for a wide range of flow Mach numbers. The effect of flow compressibility on the skin friction for a rough surface appears to be properly reflected through the use of the critical roughness Reynolds number.

Figure 2 shows a comparison of the predicted and the measured[3] roughness effects on a sharp cone with a cold wall at hypersonic speeds. The cone half angle is $\theta_c = 10.5$ deg. The freestream Mach numbers are $M = 6.6$–11.3. The Reynolds number per meter is $Re/m = 2.3 \times 10^7$–26×10^7 and the wall temperature $T_w = 0.2$–$0.38T_r$. The solid lines represent the proposed correlation for $T_w = 0.38T_r$ and the dashed lines that for $T_w = 0.2T_r$. Calculations are shown for two Re_k^c, 14 and 20, which roughly correspond to the lowest and the highest Mach numbers in the experiment. There is a higher level of spread of the data points at hypersonic speeds than at supersonic speeds. However, it appears that the data cluster around the correlations. The results in Figs. 1 and 2 indicate that the proposed correlations for surface roughness can provide satisfactory predictions for both flat-plate and sharp-cone geometries at supersonic and hypersonic speeds.

The supersonic flow over a flat plate with roughness and mass blowing has been calculated using the present prediction

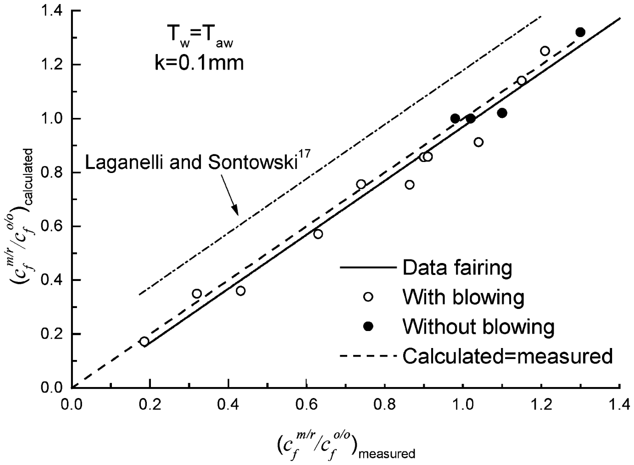

Fig. 3 Comparison of the calculated and the measured results[15] for a rough wall with and without blowing.

methodology. The data[15] reported were for $M_\infty = 2.9$, $Re_\infty/m = 3.8 \times 10^6$–$30.1 \times 10^6$, $x = 1.98$ m, and $k = 0.1$ mm. The sand-grain roughness height is $k_s = 1.22k$ and the mass-blowing rate $\dot{m} = 0$–0.1465 kg/m$^2 \cdot$ s. The Van Driest II transformation and Karman skin friction equation for incompressible turbulent boundary layers were used to calculate $c_f^{o/o}$.

For these cases with various amounts of mass blowing, the measured and the calculated skin frictions are given in Fig. 3. The comparisons are presented in the form of $(c_f^{m/r}/c_f^{o/o})_{\text{calculated}}$ vs $(c_f^{m/r}/c_f^{o/o})_{\text{measured}}$. Therefore, for a case where the calculated value agrees exactly with the data, the corresponding symbol would fall on the straight line with a slope of one, which is represented by the dashed line in Fig. 3. The symbols in Fig. 3 cluster around this dashed line, indicating that the calculated skin friction agrees well with the measurements. An error bound can be obtained using the linear regression of the symbol data points, which is shown by the solid line. For comparison, the faired line of Laganelli and Sontowski[17] has also been included in Fig. 3. On the average, the error relative to the experimental measurement is within 4% for the present calculations and is about 20% for Laganelli and Sontowski.[17] The present predictions represent better agreement with measurement.

Summary

In this Note, a new computational method for the prediction of the skin friction for high-speed turbulent boundary layers with ablation is described. The method calculates the skin friction by making use of the existing validated experimental correlations developed for surfaces with roughness and for surfaces with mass blowing, respectively. Results have been presented for high-speed turbulent boundary layers over a flat plate. When compared with the available measurement, the results obtained by using the present method agree well with the data. The computational procedures involved are algebraic operations and can readily be implemented into prediction software currently used in the research community and in industry. For instance, it may provide the much-needed wall-boundary conditions for computational fluid dynamics studies of high-speed flows with surface ablation. The formulations can also be extended to examine additional aerothermodynamics effects, such as angle of attack, rotation, and surface heat transfer at high speeds. Note that, since the methodology is not fundamentally generic, detailed validations or modifications of the correlations may be needed when the present method is applied to problems with, for example, geometric variation to the test cases.

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T. Lin
Associate Editor